$$\Phi_1(\lambda_1) = C_1 = C_2\frac{A}{\xi} - \frac{1-2\nu}{2\mu}TA - \frac{1-\nu}{2\mu}T\frac{A^3}{2\xi^2} + \frac{1-\nu}{2\mu}TA\ln(A) \quad (A = \sqrt{R^2-\xi^2}). \tag{5.2}$$

Thus, expressions (4.2), (5.1), (5.2) give a complete solution for the elastoplastic arrangement of the original problem with conditions at the contact (4.1).

An approach has been considered for solving a class of elastic deformation problems for rolled shells based on using plastic models, and by plastic here we understand existence of slippage for layers of these shells. Analysis has been carried out for the stress–strain state of these structures. It appeared that as a result of the possibility of slippage of layers a rolled shell operates better than a one-piece thick-walled tube in the sense that it is possible to redistribute more uniformly the applied load through the thickness of the structure. In particular, if the condition at the contact is taken in the form of (4.1), then it is possible to obtain an advantage in supporting capacity by almost a factor of two compared with a one-piece tube. The model provided makes it possible to consider its generalization in a number of other models taking account for example of internal friction of the material, plastic deformation of the shell layers themselves, etc.

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# ANTIPLANAR PLASTIC FLOW


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We will consider the equations describing nonsteady-state plastic flow of a Mises medium:

$$\begin{gathered}\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = \frac{\partial p}{\partial x_i} + \frac{\partial s_{ij}}{\partial x_j}, \\ s_{ij}s_{ij} = 2k_s^2, \quad 2s_{ij} = \lambda\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \\ \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0,\end{gathered} \tag{1}$$

where $u_1$, $u_2$, $u_3$ are the components of the velocity vector, p is the hydrostatic pressure, $\lambda$ is a nonnegative function, $s_{ij}$ are the components of the stress tensor deviator, $k_s$ is the yield point for pure shear, and repeating indices imply summation.

We will assume that the medium is located under conditions of antiplanar plastic flow, so that the solution of Eq. (1) will be sought in the form [1]

$$u_1 = 0,\ u_2 = 0,\ u_3 = w(x, y, t),\ p = 0. \tag{2}$$

Substituting Eq. (2) in Eq. (1) we obtain an equation describing antiplanar plastic flow:

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial x}\frac{w_x}{\sqrt{w_x^2 + w_y^2}} + \frac{\partial}{\partial y}\frac{w_y}{\sqrt{w_x^2 + w_y^2}}$$


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$$w_x = \frac{\partial w}{\partial x}, \quad w_y = \frac{\partial w}{\partial y}. \tag{3}$$

To find exact solutions of Eq. (3) we find a group of continuous transforms which that equation admits. Calculating this group by the method of [2] we find that the group is generated by the operators

$$\begin{aligned} &X_1 = x\partial_y - y\partial_x, \ X_2 = t\partial_t + w\partial_w, \\ &X_3 = t\partial_t + x\partial_x + y\partial_y, \ X_4 = \partial_t, \\ &X_5 = \partial_w, \ X_6 = \partial_x, \ X_7 = \partial_y. \end{aligned} \tag{4}$$

To find significantly differing invariant solutions of Eq. (3) we enumerate all the nonsimilar one-dimensional subalgebras for the Lie algebra with Eq. (4) as base:

a) $X_1 + \alpha X_2 + \beta X_3$;

b) $X_1 + \alpha X_3 + \beta X_5$;

c) $X_1 + \alpha X_4 + \beta X_5$;

d) $X_4 + \alpha X_5 + \beta X_6$;

e) $X_5 + \alpha X_6$;

f) $X_3 + \alpha X_5$;

g) $X_2 + \alpha X_3$;

h) $X_2 + \alpha X_6$;

i) $X_6$;

j) $X_5$ ($\alpha$, $\beta$ being arbitrary constants).

We will consider the steady-state solution invariant relative to the subgroup $X_4$. This solution satisfies the equation

$$\frac{\partial}{\partial x} \frac{w_x}{\sqrt{w_x^2 + w_y^2}} + \frac{\partial}{\partial y} \frac{w_y}{\sqrt{w_x^2 + w_y^2}} = 0, \tag{5}$$

the solution of which can be used to describe plastic flow of a long cylindrical body with arbitrary cross-sectional form under the action of loads directed along the generatrix of the cylinder and constant along that generatrix.

External loads applied to the faces of a bar are statically equivalent to a torque $G_3 = \int\int (xs_{23} - ys_{13})\, dx\, dy$. Here the z axis coincides with the cylinder axis and the axes x and y lie in the plane of the cross section, bounded by the contour $\Gamma$. Let the vector of the normal to the lateral surface have the form ($n_1$, $n_2$, 0). Since the external loads are directed along the generatrix, we have

$$n_1 s_{13} + n_2 s_{23}|_\Gamma = 0. \tag{6}$$

Consequently, it is necessary to solve the problem of Eqs. (5), (6).

After differentiation and other operations Eq. (5) reduces to the form

$$w_y^2 w_{xx} - 2w_x w_y w_{xy} + w_x^2 w_{yy} = 0, \tag{7}$$

which, as has been shown [3, p. 221] previously, admits an infinite group of Lie–Beklund transforms. Therefore, Eq. (7) can be linearized. For example, this may be done by a Legendre transform. But another method is also possible.

We introduce new unknown functions $u = w_x$, $v = w_y$, then write Eqs. (6), (7) in the form

$$v^2 u_x - 2uvu_y + u^2 v_y = 0, \quad u_y - v_x = 0; \tag{8}$$

$$n_1 u + n_2 v|_\Gamma = 0. \tag{9}$$

If we consider u, v to be components of the velocity vector, then Eq. (8) describes settled planoparallel isentropic gas flow [4, p. 256]. Boundary condition (9) implies that the gas flows in a long tube with impermeable walls.

Consequently the problem of antiplanar settled flow of a gas, Eqs. (5), (6), reduces to the problem of Eqs. (8), (9) for the gas dynamics equations. It should be noted that the

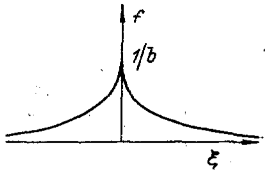

Fig. 1

problem of Eqs. (5), (6) is well developed. Powerful analytical and numerical methods are used for its solution [4, 5].

Note. Aside from boundary conditions (6), others are possible. We will consider a cylindrical body loaded on the lateral surface by forces uniformly distributed and directed along the generatrix. Then on the lateral surface [1]

$$n_1 s_{13} + n_2 s_{23}|_\Gamma = f(x, y). \tag{10}$$

We will consider some nonsteady state invariant solutions which we feel to be of definite interest. We will seek a solution in the form w = tf(x, y), so that Eq. (3) is written as: $f = \frac{\partial}{\partial x}\frac{f_x}{\sqrt{f_x^2+f_y^2}} + \frac{\partial}{\partial y}\frac{f_y}{\sqrt{f_x^2+f_y^2}}$. If we seek f in the form $f = \varphi(\theta)/r$ (where r, θ are polar coordinates), then for definition of the function φ we obtain the equation $\varphi''\varphi^2 - \varphi^3 - 2\varphi\varphi'^2 = \varphi(\varphi^2 + \varphi'^2)^{3/2}$, which can be used to describe plastic flow of a cylinder [with cross section specified by the equation r = ψ(θ)]. As the boundary condition we take Eq. (10).

Now let w = ax + f(bt − y), then Eq. (3) reduces to the ordinary differential equation $bf' = \frac{d}{d\xi}\frac{f'}{\sqrt{a^2+f'^2}}$, $\xi = bt - y$. We solve this equation, taking the arbitrary constants which appear upon integration equal to zero. As a result $\pm\left(\sqrt{1-b^2f^2} - \ln\left|\frac{1+\sqrt{1-b^2f^2}}{bf}\right|\right) = |ab|\xi$. For f > 0 the solution f = f(ξ) is shown in Fig. 1. If ξ = 0, then f = 1/b. This solution can be used to describe a plastic wave in a layer |x| ≤ h, if a tangent stress $s_{23}$ is specified on the layer boundary.

In conclusion we will present the form of other invariant solutions of Eq. (3), constructed for the subalgebras "a"-"i":

$$\text{a) } w = t^{\alpha/(2\alpha+\beta)} f(r\exp(-\beta\theta),\ t\exp(-(2\alpha+\beta)\theta)) \text{ for } 2\alpha+\beta \neq 0,$$
$$w = f(\beta\theta - \ln r,\ t)/r^{1/2} \text{ for } 2\alpha+\beta = 0,\ \beta\neq 0;$$
$$\text{b) } w = \beta\theta + f(t\exp(-\alpha\theta),\ r\exp(-\alpha\theta));$$
$$\text{c) } w = \beta\theta + f(\theta - \alpha t,\ r);\ \text{d) } w = \alpha t + f(\beta t - x,\ y);$$
$$\text{e) } w = \alpha x + f(t, y);\ \text{f) } w = \alpha\ln t + f(x/t,\ y/t);$$
$$\text{g) } w = t^{1/(2+\alpha)} f(\theta,\ r^{2+\alpha}t^{-\alpha}) \text{ for } 2+\alpha\neq 0;\ w = r^{-1/2}f(t,\theta)$$
$$\text{for } \alpha + 2 = 0;$$
$$\text{h) } w = t^{1/2} f(y,\ (\alpha/2)\ln t - x);\ \text{i) } w = f(t, y).$$

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